Twisted QR algorithms

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Motivation

Impact of the QR Algorithm

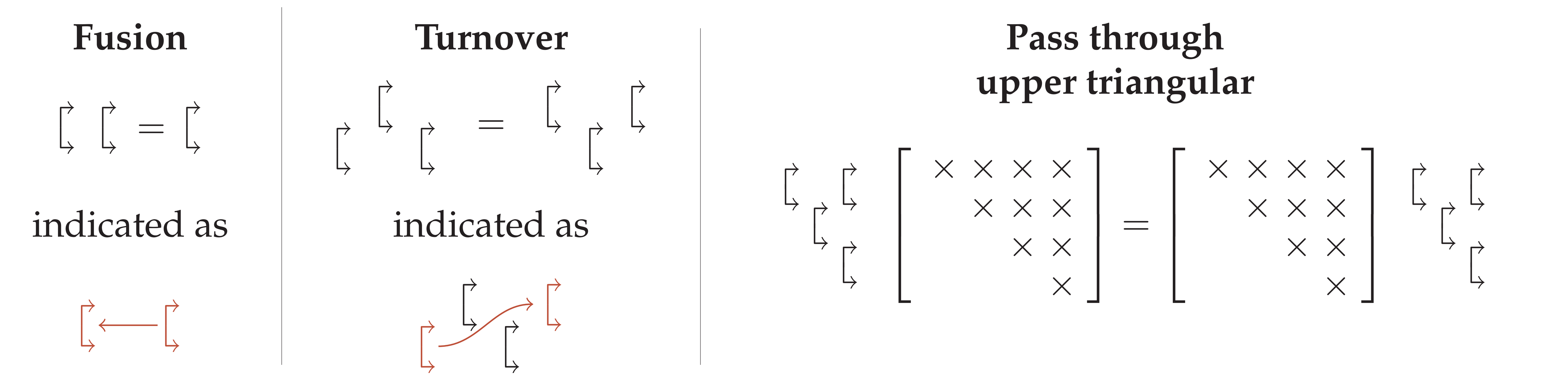
- Top 10 algorithm of the 20th century;
- 190.000 documents (scholar.google);
- SIAM/LA prizes;
- 12.500 hits on Wikipedia (last 90 days).

Motivation

- Classic QR on Hessenberg matrices.
- Why only Hessenberg matrices?
- Can we do better?

Juggling rotators

Core of the algorithm: manipulate rotators , arrows indicate the action of the rotator.



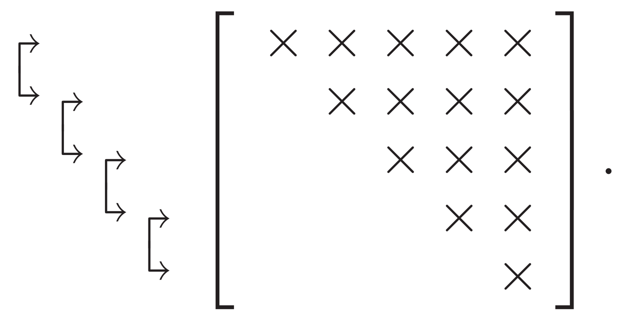
Compressed Matrices

Admissible matrix types

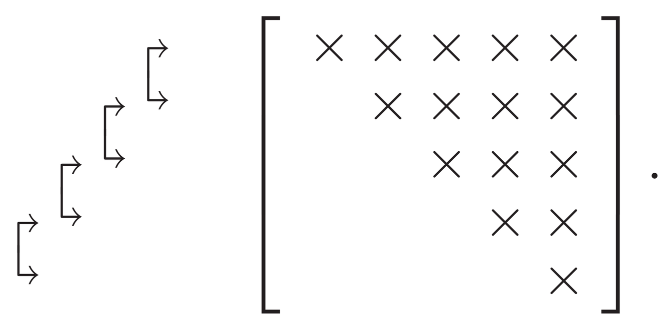
- $A = QR \in \mathbb{R}^{n\times n}$,
- Q factored in $n-1$ rotators,
- R upper triangular.
- Achievable via unitary similarity.

Examples

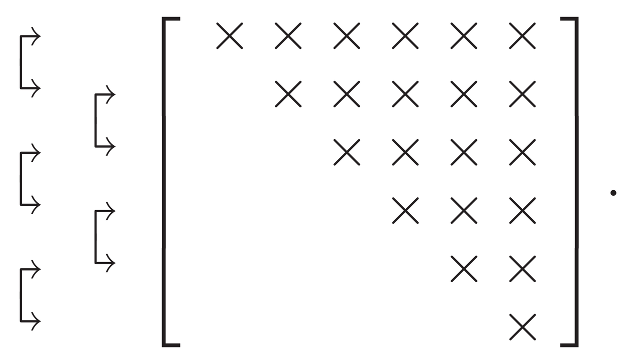
- Hessenberg (descending sequence)

$QR =$ 

- Inv. Hessenberg (ascending sequence)

$QR =$ 

- CMV shape (twisted sequence)

$QR =$ 

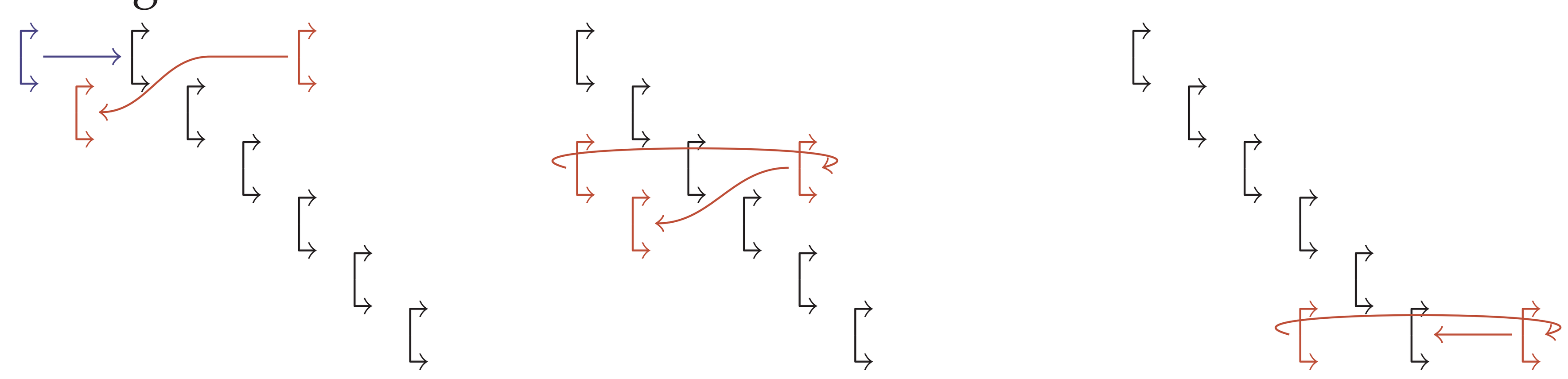
Implicit bulge chasing

Flow of the algorithm

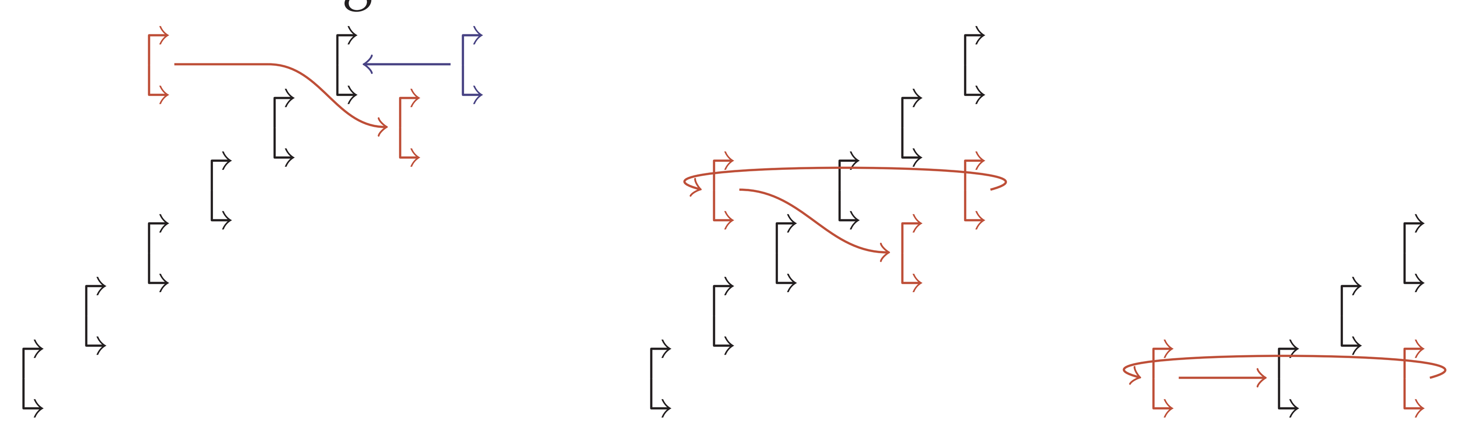
- Introduce the bulge, which equals an inappropriate rotator.
- Chase untill the end: pass through R, turnover, and do a similarity.
- At the end: fuse the rotator.
- For simplicity, we suppress the upper triangular R.

Examples

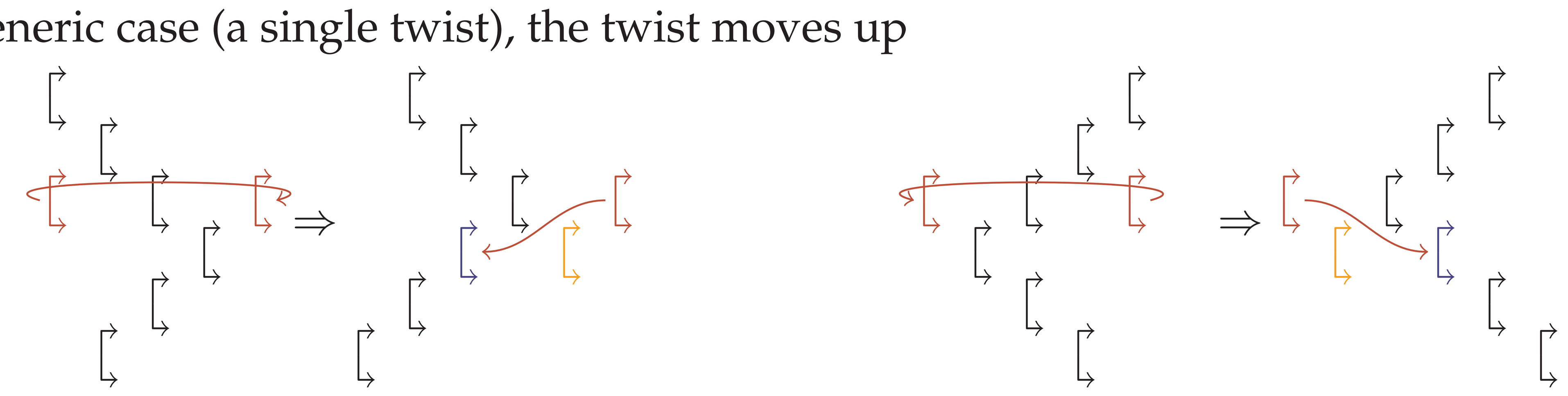
- Hessenberg



- Inverse Hessenberg



- Generic case (a single twist), the twist moves up



Extensions

Extensions

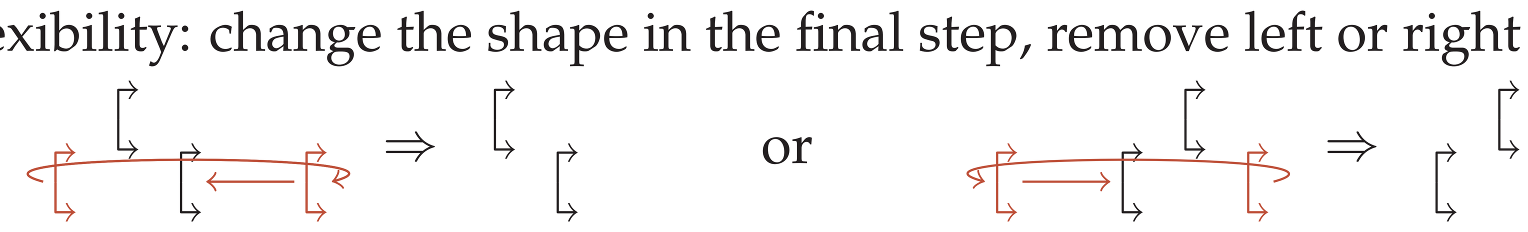
- Multishift.
- Aggressive early deflation.
- Piping tightly packed bulges.
- Generalized eigenvalue problem: n^3 admissable pencil types (A, B).

Future tasks

- Fortran implementation.
- Predicting the desired shape.

Properties

- Flexibility: change the shape in the final step, remove left or right rotator



- Convergence determined by shifts μ_j and shape (powers). Subspace iteration driven by

$$(A-\mu_1)(A-\mu_2)\cdots(A-\mu_k)(A-\mu_{k+1})^{-1}(A-\mu_{k+2})^{-1}\cdots(A-\mu_m)^{-1}.$$

- Deflation: set $G_\tau = I$. Gives error bound for (λ, x, y) eigentriple, $\exists\mu$ such that

$$\frac{|\mu-\lambda|}{|\lambda|} \le \frac{\|I-G_\tau\|_2}{x^H y} + \mathcal{O}\left(\|I-G_\tau\|_2^2\right).$$

References

[1] T. Mach and R. Vandebril, *On deflations in extended QR algorithms*, SIAM J. Matrix Anal. Appl., 35 (2014), pp. 559–579.

[2] R. Vandebril, *Chasing bulges or rotations? A metamorphosis of the QR-algorithm*, SIAM J. Matrix Anal. Appl., 32 (2011), pp. 217–247.

[3] R. Vandebril and D. S. Watkins, *An extension of the QZ algorithm beyond the Hessenberg-upper triangular pencil*, Electron. Trans. Numer. Anal., 40 (2012), pp. 17–35.

[4] ——, *A generalization of the multishift QR algorithm*, SIAM J. Matrix Anal. Appl., 33 (2012), pp. 759–779.

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